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Dependence of widths of the integer quantum Hall plateau on quantum lifetime

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Abstract

In this report we demonstrate that the width of the spin-split plateaus of the integer quantum Hall effect are determined by the quantum lifetime of the electrons via the exchange interaction. The widths of the adjacent Landau-split plateaus are a consequence of the pronounced changes of the spin-split plateaus. In contrast, the transport lifetime and thus the electron mobility are not directly related to the width of the quantum Hall plateaus.

1. Introduction

The integer quantum Hall effect (QHE) [1, 2] has been the subject of intensive experimental and theoretical research over the last two decades. Nevertheless, a comprehensive theoretical treatment describing the integer QHE is still not available. The majority of the existing theoretical treatments deal with the transition region between adjacent plateaus and the corresponding localization length [3]. Less attention has been paid to the width of the quantized plateaus, which should be a good indicator of the stability of the quantum Hall states [4]. After the discovery of the QHE, a linear decrease in the plateau widths with increasing current [5] and increasing temperature [6] was measured. Furthermore, a direct correlation with electron mobility as the characteristic transport parameter of the electrons was reported [7, 8]. The width should decrease linearly with increasing zero-field mobility. Recently, the influence of special scattering mechanisms on the plateau width has also been investigated theoretically [9].

The plateaus are usually explained in terms of the localization model [3], which assumes the existence of localized and extended states in the tails and centres of the disorder-broadened Landau bands, respectively. In this model the widths of the plateaus depend on the ratio of localized versus extended states. The localization model is often used to explain the mobility dependence of the plateaus' widths. With increasing electron mobility the delocalized range of the density of states (DOS) broadens and, as consequence, the localized range of the DOS and thus the widths of the quantum Hall plateaus decrease [8, 10].

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In this paper we show that this simplified picture must be applied with great care. We report on systematic studies, carried out at a low but fixed temperature, of the dependence of the plateau widths on the characteristic lifetimes. The relative plateau widths at a fixed temperature are determined primarily by these lifetimes, which in general may be temperature dependent. Other studies of temperature-dependent effects, especially those on the scaling behaviour, should carefully consider this. In our paper we distinguish between the transport lifetime τ_t, which determines the electron mobility μ described by

$$\mu = \frac{e\tau_t}{m^*} \tag{1}$$

where e is the electron charge, m^* the effective electron mass and τ_q the quantum lifetime. Whereas the quantum lifetime counts every scattering event equally, the transport lifetime favours large-angle scattering over small-angle scattering [11, 12]. The quantum lifetime was extracted from the amplitude of the Shubnikov–de Haas (SdH) oscillations [13] and the transport lifetime from the longitudinal resistance without a magnetic field.

Significantly, the width of the spin-split $i = 3$ plateau increases continuously with increasing quantum lifetime. We show that this broadening can be attributed to the quantum lifetime via the effective g-factor and the exchange interaction of the electrons in different spin-split Landau levels. Therefore the widths of the adjacent Landau-split plateaus are a consequence of the pronounced changes of the spin-split plateaus. On the other hand, the transport lifetime, which determines the electron mobility, does not allow precise prediction of the widths of quantum Hall plateaus.

2. Sample structure

For the purpose of our study it was absolutely essential to compare the data of the different samples used for the same 2D carrier density, since both the quantum lifetime and transport lifetime depend on carrier density due to free-carrier screening of the ionized impurity potential [14]. We have thus chosen a so-called inverted modulation doped structure, where the doping is beneath the quantum well and a top Schottky contact. Even though the electron mobility in these structures is generally lower than that in ‘normal’ modulation doped structures (i.e. modulation doping above the quantum well) the electron density could be varied and tuned to be exactly the same for all samples via the voltage applied to the top Schottky gate.

The modulation-doped GaAs/($Al_{0.3}Ga_{0.7}$)As structures were grown by molecular beam epitaxy with a 10 nm thick quantum well in which the electron density could be varied between zero and 4×10^{11} cm^{-2}. A standard Hall-bar geometry with ohmic contacts made of an alloy of Ni–AuGe–Ni was used. The samples that were studied were grown under nominally identical growth conditions, but with systematic changes in the growth interrupts before and after the quantum well, which determines the properties (roughness, impurity incorporation) of the respective interface [15].

3. Experimental results

The measurements were performed in a top-loading dilution refrigerator at a temperature of 30 mK. A carrier density of 2×10^{11} cm^{-2} was always used for the measurements. At this carrier density, depending on the interface properties, the quantum lifetime of the samples has values between 0.15 and 1 ps and the transport lifetime between 0.2 and 1.8 ps. As the widths of the Hall plateaus, we determine the region of magnetic field for which

$$\rho_{xy} = h/(ie^2) \pm 0.1\,\mathrm{k\Omega} \tag{2}$$

where h is Planck’s constant and i is an integer plateau number.

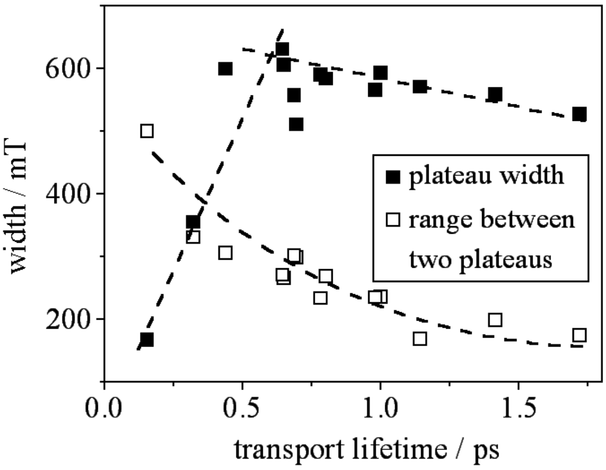

Figure 1. The width of the $i = 4$ plateau and the magnetic field range between the $i = 4$ and 3 plateaus versus transport lifetime, which determines the electron mobility. The dashed curves are guides for the eye.

To illustrate the problem of the simple model used to explain the plateau width, we consider the widths of the $i = 4$ plateau and the transition range between plateaus 3 and 4, and their dependence on transport lifetime and hence mobility (figure 1). Starting with a plateau width of 170 mT for a transport lifetime of 0.2 ps, the width rises by a factor of four for a corresponding lifetime of 0.7 ps. The short lifetimes represent the on-set of the QHE, in which the first plateaus are formed and a deviation from the classical straight line occurs. As a consequence, the width of the $i = 4$ plateau increases over this range. For transport lifetimes larger than 0.7 ps, the widths of the plateau reveal the predicted decrease.

However, if the magnetic field range between two adjacent plateaus is also considered then one can see an apparent contradiction. For transport lifetimes where the width of the $i = 4$ plateau decreases, the magnetic field range between the $i = 4$ and 3 plateaus also decreases (figure 1, open symbols). This contradiction casts doubt on the simple explanation that the width of the plateaus is a consequence of an enhanced delocalized energy interval of the DOS due to the longer lifetime. In this model the assumption is made that the width and the shape of the DOS remain unchanged. We believe that this assumption is not valid.

To resolve the contradiction we turn our attention to the plateau with filling factor three, which exhibits by far the largest changes of the plateau width for different samples, as shown in figure 2. For samples with small lifetimes an $i = 3$ plateau does not exist, whereas for samples with long lifetimes well developed and flat $i = 3$ plateau are seen. In figure 3 the changes in the $i = 4$ plateau, now plotted as a function of the quantum lifetime, are compared with the quantum lifetime dependence of the width of the $i = 3$ plateau.

The broadening of the $i = 4$ plateaus only takes place in the range of small lifetimes ($\tau_q < 0.4$ ps), where no $i = 3$ plateau exists. For quantum lifetimes larger than 0.4 ps, the $i = 3$ plateau develops and the widths of the two plateaus show an opposite dependence. The continuous and pronounced increase in the width of the $i = 3$ plateau is opposite to the decrease in the width of the $i = 4$ plateau. Due to the broadening of the $i = 3$ plateaus, a smaller magnetic field range is available for the neighbouring plateaus. Therefore the increase in the width of the $i = 3$ plateau is responsible for the slight decrease in the width of the $i = 4$ plateau.

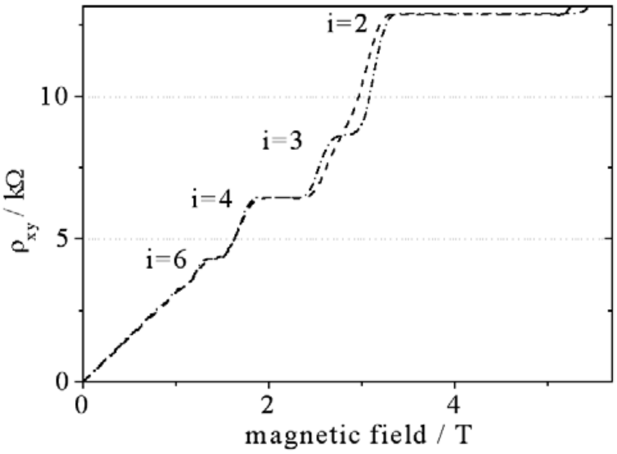

Figure 2. Typical QHE curves for three samples exhibiting pronounced changes for the $i = 3$ plateau. The samples were grown with different growth interrupts, resulting in different lifetimes.

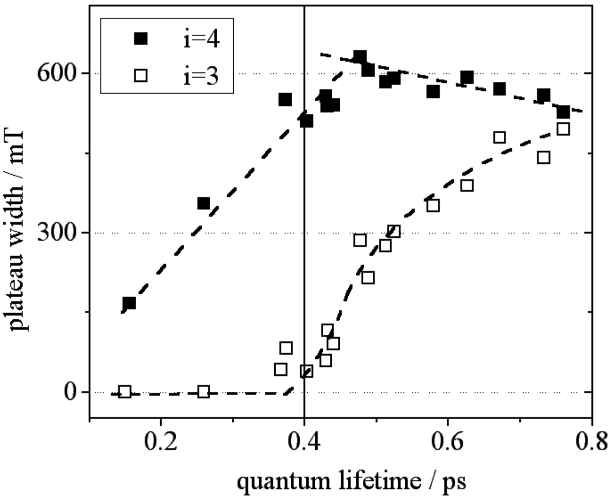

Figure 3. Plateau width versus quantum lifetime for the $i = 3$ and 4 plateaus.

4. Physical causes for the width of the plateaus

In the following, the microscopic origin for the behavior of the spin-split $i = 3$ plateau is discussed. The spin-splitting is determined by the Zeeman energy E_S,

$$E_S = sg^*\mu_B B \tag{3}$$

where s is the spin quantum number $(\pm\frac{1}{2})$, μ_B is the Bohr magneton, and g^* is the enhanced effective g-factor, which is composed of the bare g-factor and the contribution due to the exchange interaction between electrons in different levels such that

$$g^*\mu_B B = g\mu_B B + E_{ex}, \tag{4}$$

where E_{ex} is the exchange energy.

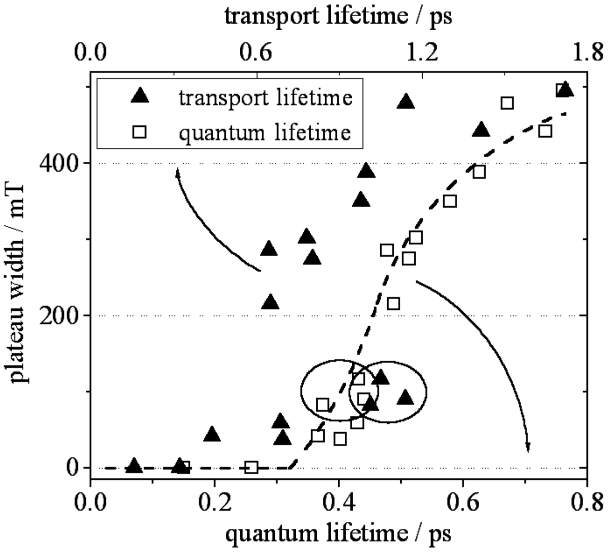

Figure 4. The width of the $i = 3$ plateau versus quantum lifetime (open squares) and transport lifetime (solid triangles). Three samples that were grown with lower arsenic pressure are marked with circles.

The spin splitting due to the bare g-factor for all magnetic fields is so small that an $i = 3$ plateau is not to be expected ($g = 0.5$). This is made possible only by the exchange interaction of the electrons in the two levels, whereby the effective g-factor (g^*) is increased. Due to the dependence of the exchange energy on filling factor, an oscillating effective g-factor arises as a function of the filling factor, with a maximum between two spin-split levels and a minimum between two Landau-split levels [16].

The magnitude of the exchange energy determines the maximum magnetic field range for a spin-split quantum Hall plateau. The exchange energy is determined by the difference in population of the two spin states of a given Landau level:

$$E_{\mathrm{ex}} \propto n_{N\uparrow} - n_{N\downarrow}. \tag{5}$$

The difference in the relative populations in turn depends on the broadening of the Landau levels and increases for narrower Landau levels [17].

In the ideal case without disorder and inelastic scattering, discrete Landau levels would be expected. With increasing scattering, the Landau levels effectively broaden and the relative population difference decreases. Consequently, the exchange energy between the electrons and, accordingly, the effective g-factor are reduced. Therefore the magnetic field range for the spin-split Landau level is reduced and, together with just a slight change in the slope between the plateaus, the width of the spin-split plateau decreases. Thus its width is linked directly to the broadening of the effective Landau levels.

The collision broadening of the Landau levels is related to the quantum lifetime [11]:

$$\Gamma_{\mathrm{q}} = \frac{\hbar}{2\tau_{\mathrm{q}}}. \tag{6}$$

In contrast with the quantum lifetime, the transport lifetime, which determines the mobility, represents just a fraction of the actual number of collisions. Thus the transport lifetime is not directly linked to the width of the $i = 3$ plateaus. This is confirmed by the data shown in figure 4, where the width of the $i = 3$ plateau is plotted versus quantum lifetime and transport lifetime, respectively. While we obtain a continuous and monotonic dependence of the plateau width on quantum lifetime, this is not the case for the transport lifetime, because samples that were grown with lower arsenic pressure (marked with a circle), for example, do not fit in the continuous increase of the other samples. Thus the quantum lifetime determines the width of the $i = 3$ plateau and hence that of the neighbouring plateaus.

5. Conclusions

In conclusion, we have demonstrated that the width of the spin-split $i = 3$ quantum Hall plateau is determined by the quantum lifetime of the electrons. The respective changes in the width of the $i = 3$ plateau are responsible for the dependence of the plateaus of neighbouring filling factors. In contrast, the transport lifetime, which only counts a fraction of all the scattering events, is not directly related to the broadening of the Landau levels and the width of the integer quantum Hall plateaus. As a consequence, our results demonstrate that the width of the $i = 3$ quantum Hall plateau is a quantity that is characteristic of the quantum well interfaces' quality, as determined by the roughness and impurity incorporation.

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